Numerical Simulation of Shock Wave Bubble Interaction using laser-induced Cavitation Bubbles [1]

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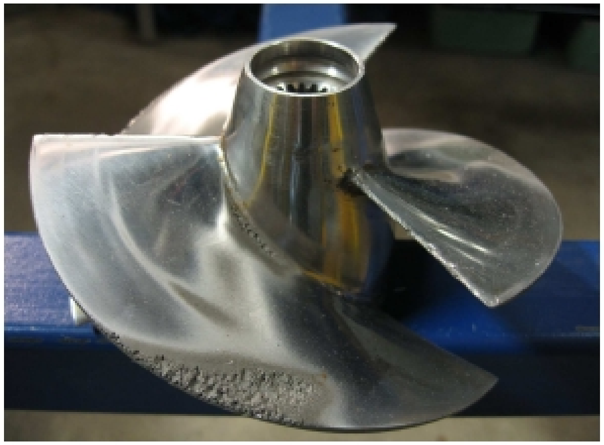
[1] DFG-CNRS FOR 563: Micro-Macro Modelling and Simulation of Liquid-Vapor Flows

Outline

- Motivation
- Mathematical Model
- Numerical Discretization
  - Finite Volume Scheme
  - Modified Ghost Fluid Method in 2D
  - Level Set Evolution
- Shock Bubble Interaction
  - Experimental & Computational Setup
  - Qualitative & Quantitative Comparison
- Conclusion

Cavitation in Liquids

- Phenomena
  - Formation of vapor bubbles if pressure drops below vapor pressure
- Problem: Material damage
  - at ship propellers
  - in oil pumps and liquid injectors
  - in medical applications, e.g. lithotripsy



- Basic experiments with laser-induced cavitation bubbles
  - Far from and near solid boundaries of different stiffness
  - Interaction of lithotripter shock wave with bubbles
    * Bubble oscillation, splitting and coalescing
    * Formation of liquid jets
    * Swarm of bubbles

Challenges for Numerical Simulation

- Physical Modelling
  - Shocks can be formed in the gas as well as in the liquid
  - Extreme changes of state due to compression and shocks in gas and liquid
  - Inhomogeneous fields even in small bubbles due to strong wave processes

  $\Longrightarrow$ Compressibility of gas **and** liquid

- Numerical Discretization
  - Extreme change of bubble size due to bubble oscillation
  - Instability at phase boundary due to extreme difference in acoustic impedance
  - Small time steps due to fast bubble collapse
  - Highly instationary dynamics due to complicate wave interaction

  $\Longrightarrow$ High resolution and accuracy in space and time

- Validation/Experiments
  - Determine gas state inside bubble
  - Measurement of physical quantities (high speed photography, PIV, infrared camera)

  $\Longrightarrow$ Quantitative comparison

Mathematical Model

- **Assumption**:

  high speed flows and short observation times
  $\rightsquigarrow$ two phases are immiscible
  $\rightsquigarrow$ no phase transition

- **Compressible 2D Euler equations in cylindrical coordinates for conserved quantities**:

$$\begin{aligned}
&\frac{\partial}{\partial t}(r\rho) + \frac{\partial}{\partial r}(r\rho\, u_r) + \frac{\partial}{\partial z}(r\rho\, u_z) = 0,\\
&\frac{\partial}{\partial t}(r\rho\, u_r) + \frac{\partial}{\partial r}\left(r\left(\rho\, u_r^2 + p\right)\right) + \frac{\partial}{\partial z}\left(r\left(\rho\, u_r u_z\right)\right) = p,\\
&\frac{\partial}{\partial t}(r\rho\, u_z) + \frac{\partial}{\partial r}\left(r\left(\rho\, u_r u_z\right)\right) + \frac{\partial}{\partial z}\left(r\left(\rho\, u_z^2 + p\right)\right) = 0,\\
&\frac{\partial}{\partial t}(r\rho\, E) + \frac{\partial}{\partial r}\left(r\, u_r\left(\rho\, E + p\right)\right) + \frac{\partial}{\partial z}\left(r\, u_z\left(\rho\, E + p\right)\right) = 0
\end{aligned}$$

- **Stiffened gas law**:

$$p(\rho, e, \varphi) = (\gamma(\varphi) - 1)\,\rho\, e - \gamma(\varphi)\,\pi(\varphi)$$

with γ the specific heat ratio, π the minimal pressure and φ the level set function

Evolution of Phase Boundary by Level Set Function

- **Level Set Function**: $\varphi < 0$ (gas), $\varphi = 0$ (interface), $\varphi > 0$ (liquid),

- **Evolution equation**: (no mass transfer)

$$\frac{D\,\varphi}{D\,t} \equiv \frac{\partial\,\varphi}{\partial\,t} + \boldsymbol{u}\cdot\nabla\varphi = 0$$

- **Initialization**: Distance function

$$\varphi(\boldsymbol{x},t) = \begin{cases} -d(\boldsymbol{x}) & ,\boldsymbol{x}\in \text{gas} \\ 0 & ,\boldsymbol{x}\in\Gamma \\ +d(\boldsymbol{x}) & ,\boldsymbol{x}\in \text{liquid} \end{cases}, \quad \text{with} \quad d(\boldsymbol{x}) = \inf_{\boldsymbol{y}\in\Gamma}\|\boldsymbol{x}-\boldsymbol{y}\|_2$$

- **Reinitialization**: (Sussman et al.)

$$\frac{\partial\tilde{\varphi}}{\partial\tau} = \text{sgn}(\tilde{\varphi})\;(1-\|\nabla\tilde{\varphi}\|_2) \quad \text{resp.}$$

$$\frac{\partial\,\tilde{\varphi}}{\partial\,\tau} + a\cdot\nabla\tilde{\varphi} = \text{sgn}(\tilde{\varphi}), \quad a = \text{sgn}(\tilde{\varphi})\,\nabla\tilde{\varphi}/\|\nabla\tilde{\varphi}\|_2$$

Discretization of the Finite Volume Scheme

- **Finite Volume Discretization**:

$$\frac{\partial (r\boldsymbol{U})}{\partial t} + \frac{\partial (r\boldsymbol{F}_r)}{\partial r} + \frac{\partial (r\boldsymbol{F}_z)}{\partial z} = \boldsymbol{S}$$

  - Time Evolution of conserved quantities:

$$\begin{aligned} \widetilde{\boldsymbol{U}}_i^{n+1} = \boldsymbol{U}_i^n \quad &- \quad \frac{\Delta t \Delta z}{\overline{V}} \left(r_{i+\frac{1}{2}} \boldsymbol{F}^{n,-}_{r_{i+\frac{1}{2}}, z_i} - r_{i-\frac{1}{2}} \boldsymbol{F}^{n,+}_{r_{i-\frac{1}{2}}, z_i} \right) \\ &- \quad \frac{\Delta t \Delta r}{\overline{V}} \left(r_i \boldsymbol{F}^{n,-}_{r_i, z_{i+\frac{1}{2}}} - r_i \boldsymbol{F}^{n,+}_{r_i, z_{i-\frac{1}{2}}} \right) + \frac{\Delta t \Delta r \Delta z}{\overline{V}} \widetilde{\boldsymbol{S}}_i^n \end{aligned}$$

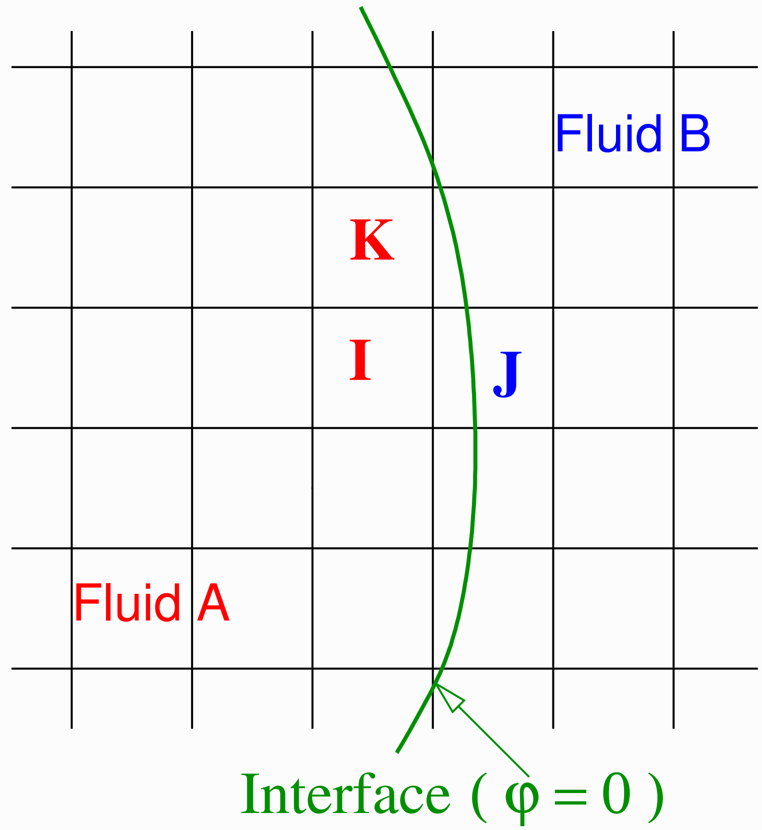
with

$$\Delta z := z_{i+\frac{1}{2}} - z_{i-\frac{1}{2}}\,, \quad \Delta r := r_{i+\frac{1}{2}} - r_{i-\frac{1}{2}}$$

$$r_i := \frac{1}{2}\left(r_{i+\frac{1}{2}} + r_{i-\frac{1}{2}} \right)\,, \quad \widetilde{\boldsymbol{S}}_i^n = (0,\ p_i^n,\ 0,\ 0)^T$$

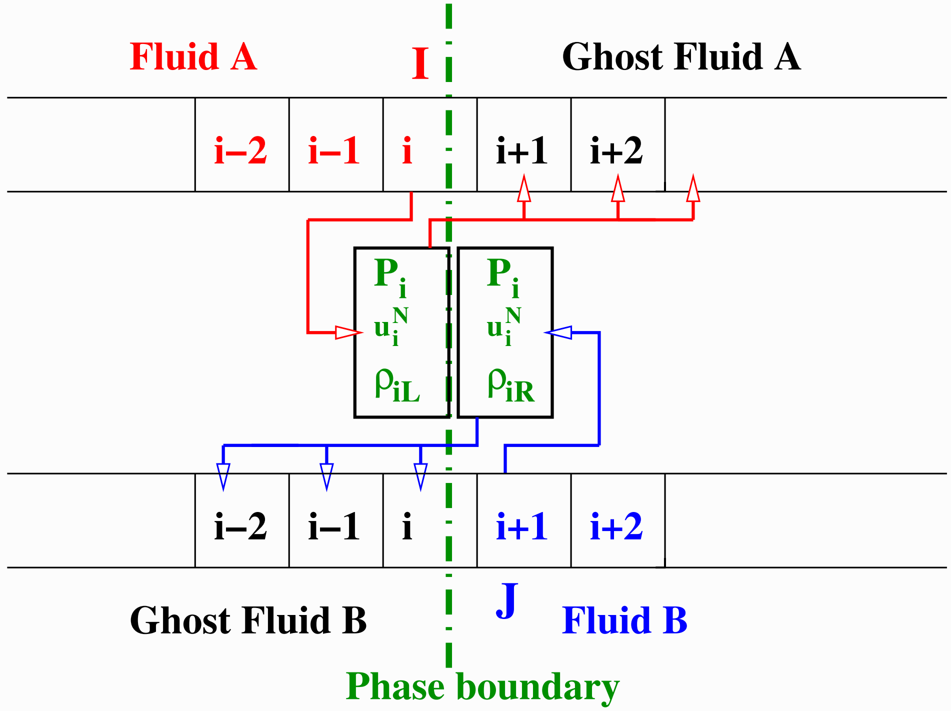
$$\overline{V} := \int_V r\, dr dz$$

Numerical Flux Computation



- Numerical Flux at the cell interface Γ_{IK} with $\varphi_I \times \varphi_K > 0$:
  - Compute 2nd order reconstruction of primitive variables: $\boldsymbol{W}_{IK}^{\pm}$
  - Solve single-phase Riemann problem: $\overline{\boldsymbol{W}}_{IK} = R(\xi = 0, \boldsymbol{W}_{IK}^{-}, \boldsymbol{W}_{IK}^{+})$
  - Evaluate flux: $\boldsymbol{F}_{IK}^{n,-} = \boldsymbol{F}_{IK}^{n,+} = \boldsymbol{F}(\overline{\boldsymbol{W}}_{IK})$

Numerical Flux at Phase Boundary: Modified GFM



- Numerical Flux at the cell interface Γ_{IJ} with $\varphi_I \times \varphi_J < 0$
  - Solve a *two-phase* Riemann problem according to Wang, Liu and Khoo with the states $\boldsymbol{W}_I = (\rho_I, u_I^N, p_I)^T$ and $\boldsymbol{W}_J = (\rho_J, u_J^N, p_J)^T$
    $\rightsquigarrow$ interfacial states: $\boldsymbol{W}_{IJ}^{-} := (\rho_{iL}, u_i, p_i)^T$ and $\boldsymbol{W}_{IJ}^{+} := (\rho_{iR}, u_i, p_i)^T$
  - Define states in ghost cells (boundary cells of fluid A and B) by interfacial states:
    ghost fluid A: $\boldsymbol{W}_j := \boldsymbol{W}_{IJ}^{-}$, $j > i+1$
    ghost fluid B: $\boldsymbol{W}_j := \boldsymbol{W}_{IJ}^{+}$, $j < i-1$
  - Solve two *single-phase* Riemann problems for fluid A and B
    $\rightsquigarrow$ two numerical fluxes $\boldsymbol{F}_{IJ}^{n,\pm}$ at the cell interface next to the phase boundary

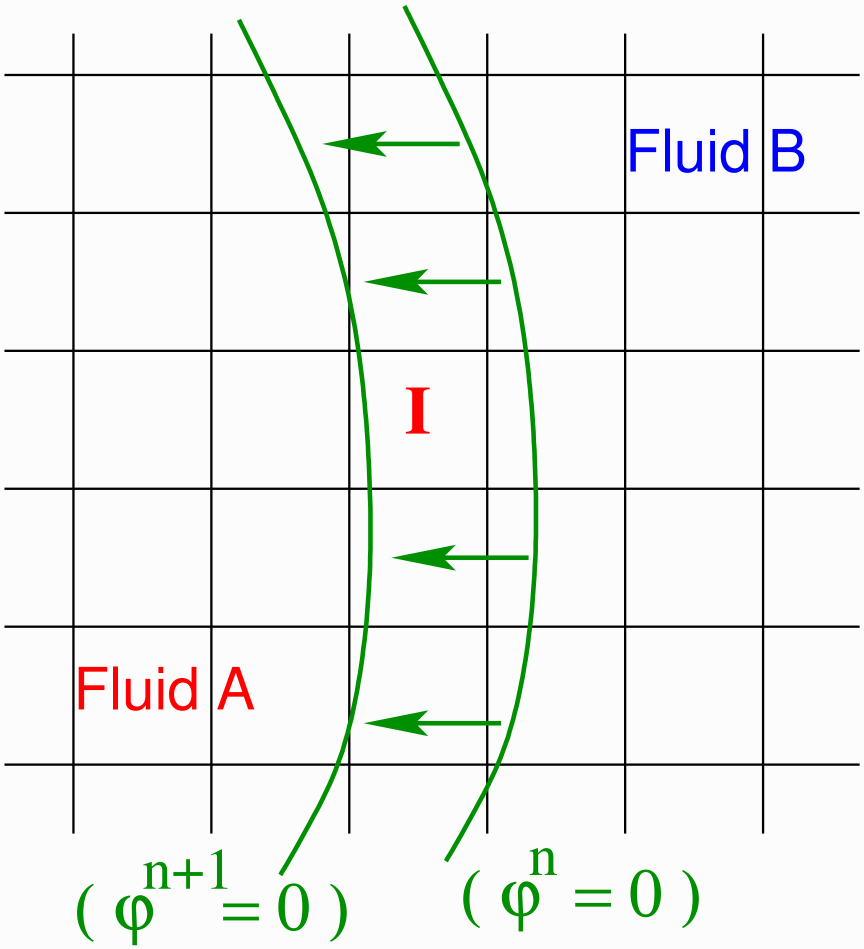
Level Set Evolution

- Transport of the level set: 2nd order compact WENO discretizaton
- Reinitialization of the level set: 2nd order ENO-type discretizaton
- Phase change when $\varphi^{n+1} \times \varphi^{n} < 0$:
  - Density correction (Farhat)

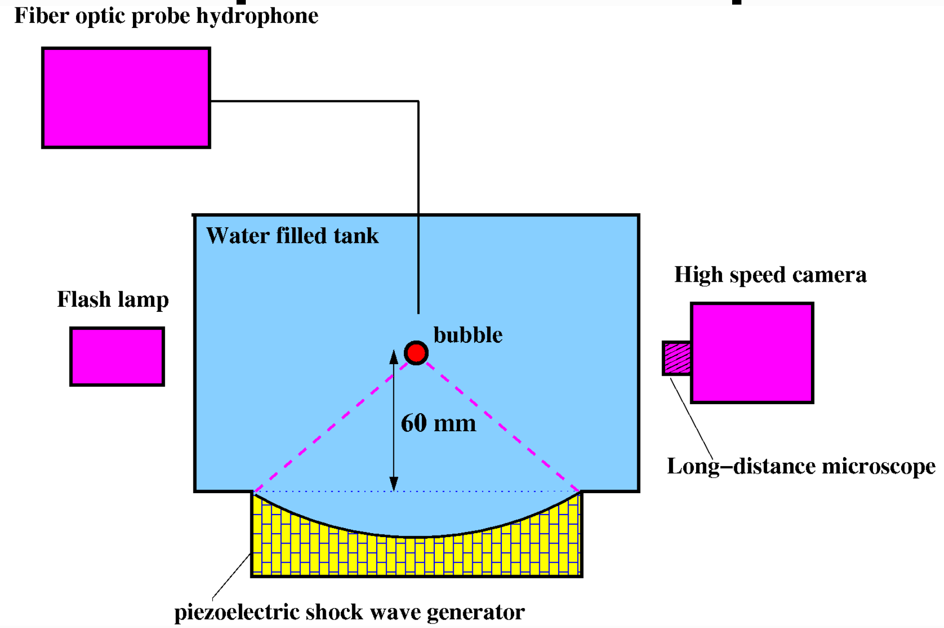
$$\tilde{\rho}^{n+1} \leftarrow \begin{cases} \rho_{iR}, & \text{if } I \text{ goes to fluid B} \\ \rho_{iL}, & \text{if } I \text{ goes to fluid A} \end{cases} .$$

  - Energy correction with constant velocity and pressure (Barberon)

$$\begin{aligned} U^{n+1} &= \rho^{n+1} \cdot \left(1,\ u^{n+1},\ E^{n+1}\right)^T \\ &\to p\left(\rho^{n+1},\ e^{n+1},\ \varphi^{n}\right) \\ &\to \tilde{e}\left(\tilde{\rho}^{n+1},\ p^{n+1},\ \varphi^{n+1}\right) \to \\ \tilde{U}^{n+1} &= \tilde{\rho}^{n+1} \cdot \left(1,\ u^{n+1},\ \tilde{E}^{n+1}\right)^T \end{aligned}$$



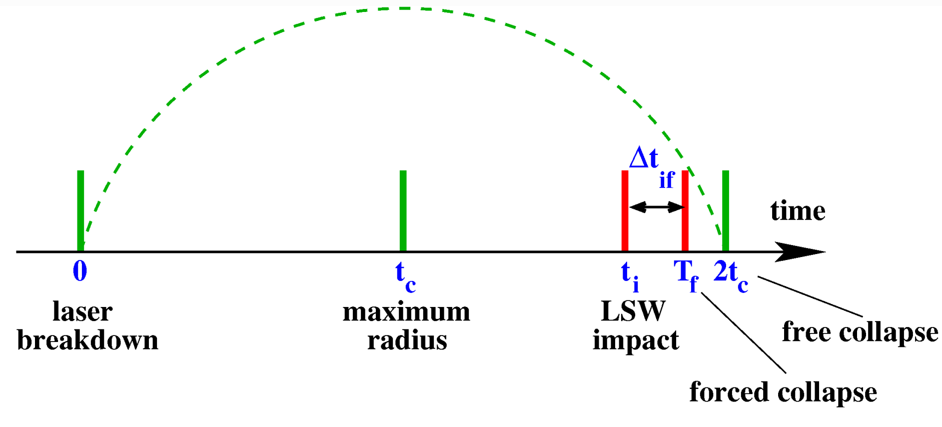
Experimental Setup[2]



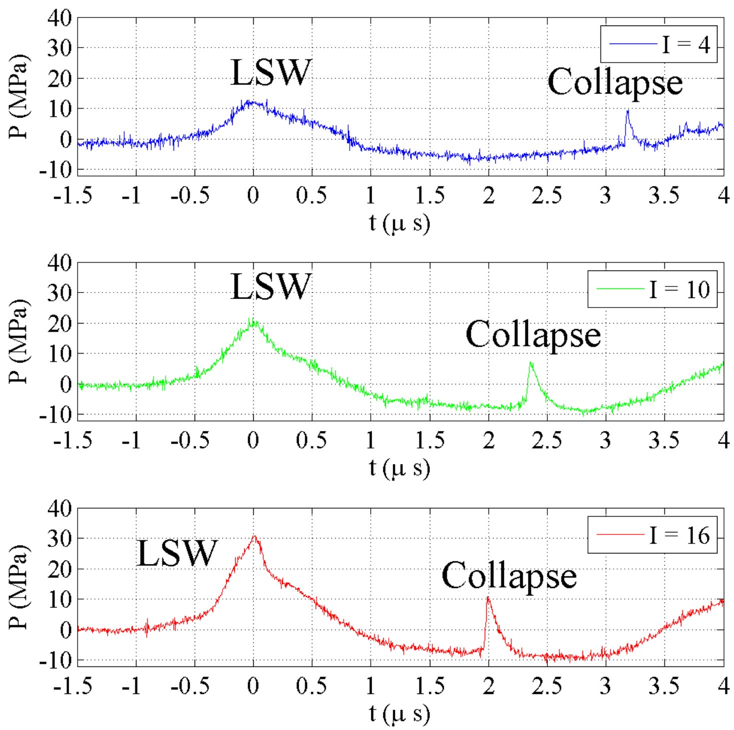
- Laser-induced cavitation bubble
- High speed camera (100 million frames per second)
- Lithotripter shock wave (LSW) generator (20 MPa $< P <$ 110 MPa)
- Fiber optic probe hydrophone at 1.88 mm above the bubble center.

[2]M. Alizadeh. Experimental investigation of shock wave - bubble interaction. PhD thesis. Georg-August-Universität Göttingen, 2010.

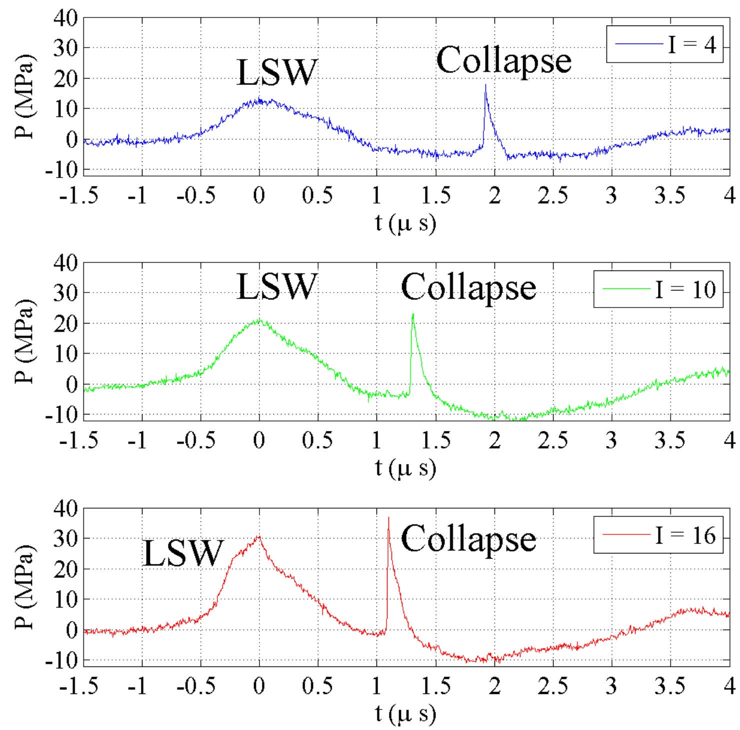
Experimental Results: Pressure Measurements



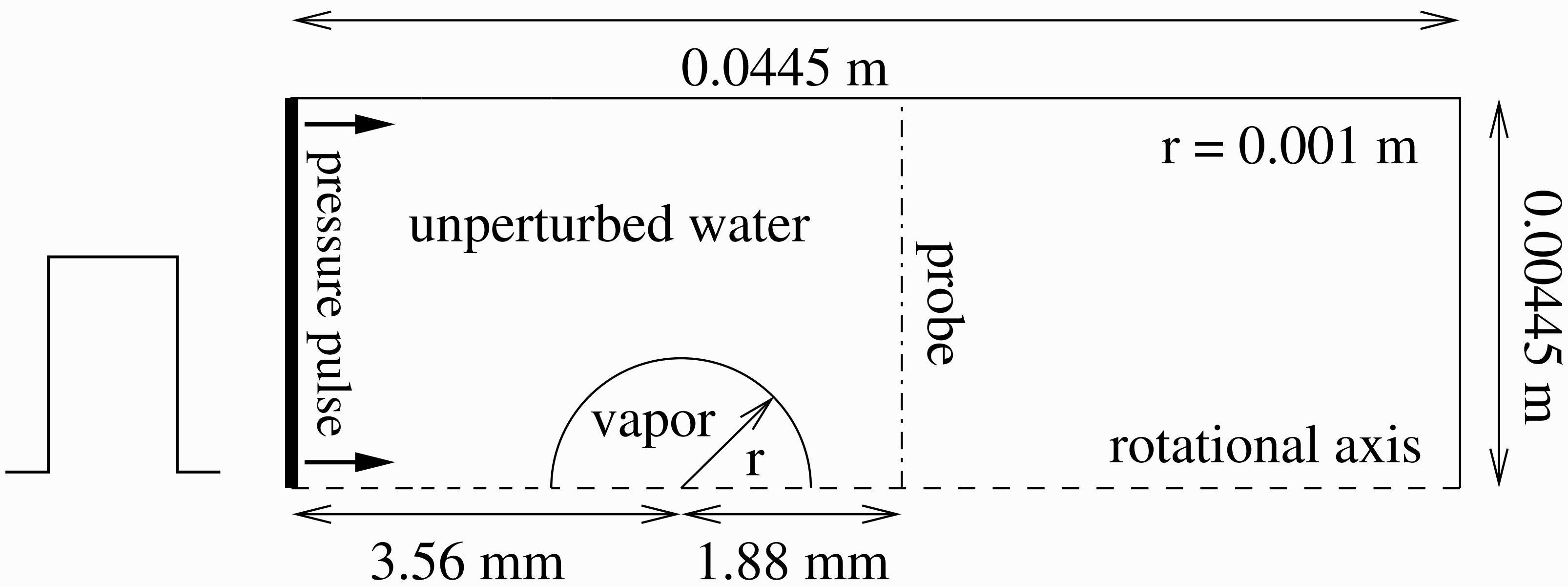
Interaction when $r = 0.39$ mm



Interaction when $r = 0.33$ mm



Computational Setup



Step 1: collapse of the bubble at its maximal radius in free field

Step 2: generation of the pressure pulse at the left boundary at $t = 44.10\,\mu$s during $0.42\,\mu$s

Step 3: interaction between the bubble and the pressure pulse in free field

Material parameters

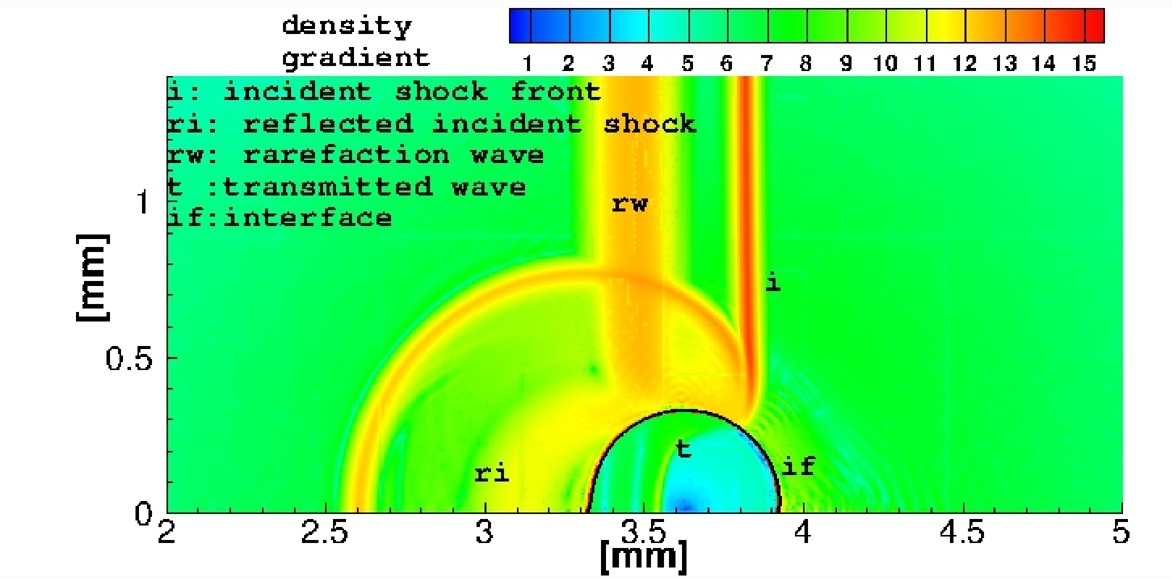
		Vapor	Water
γ		1.4	1.1
π	$[Pa]$	0	2045354545

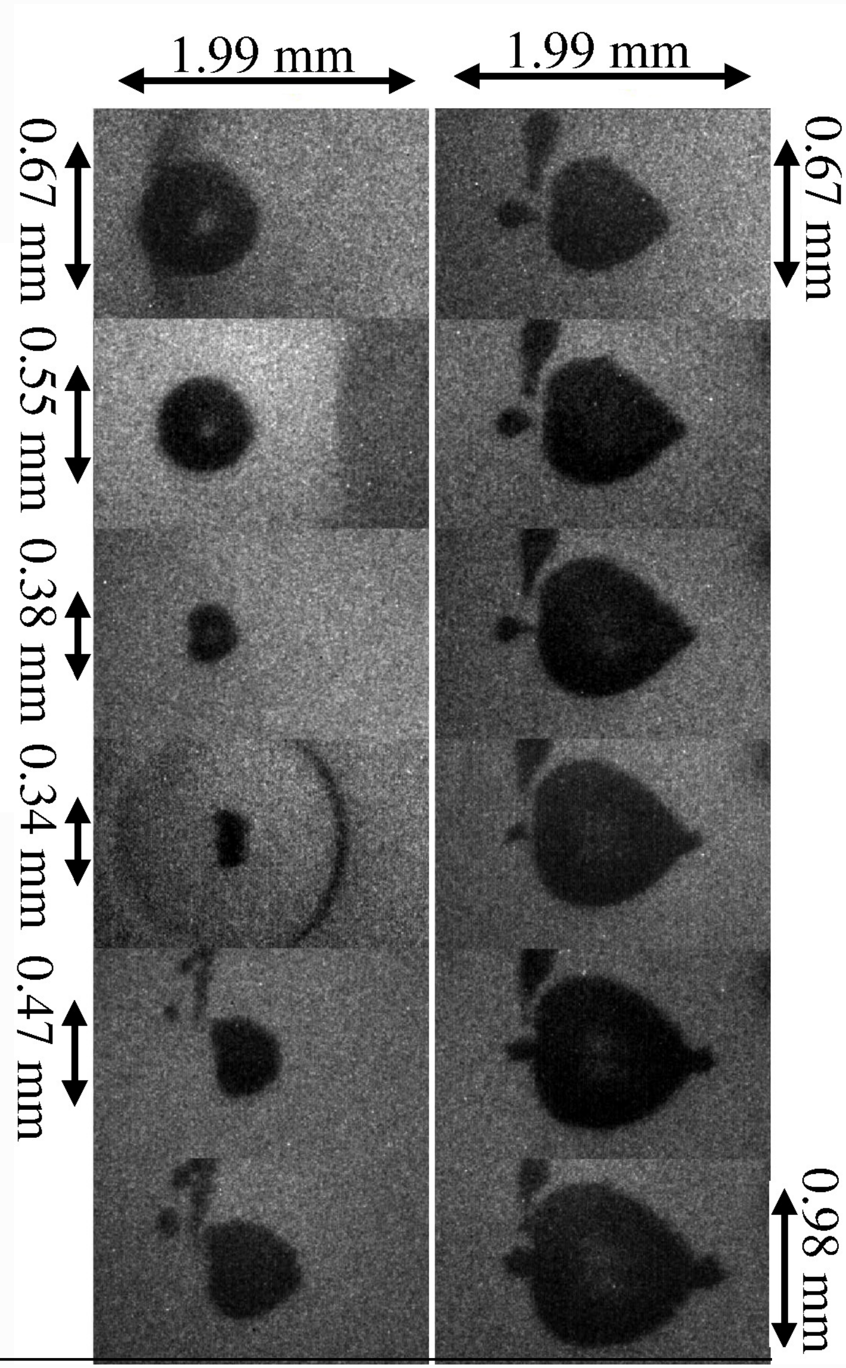
Initial conditions for $M_s = 1.01$

		vapor	unperturbed water	pertubed water
ϱ	$[kg/m^3]$	0.02782	1000	1019.1
v_x	$[m/s]$	0	0	28.43
p	$[Pa]$	2339	100000	43171429
T	$[K]$	293	293.15	293.7

Numerical Result: Qualitative Comparison

- Interaction with the bubble



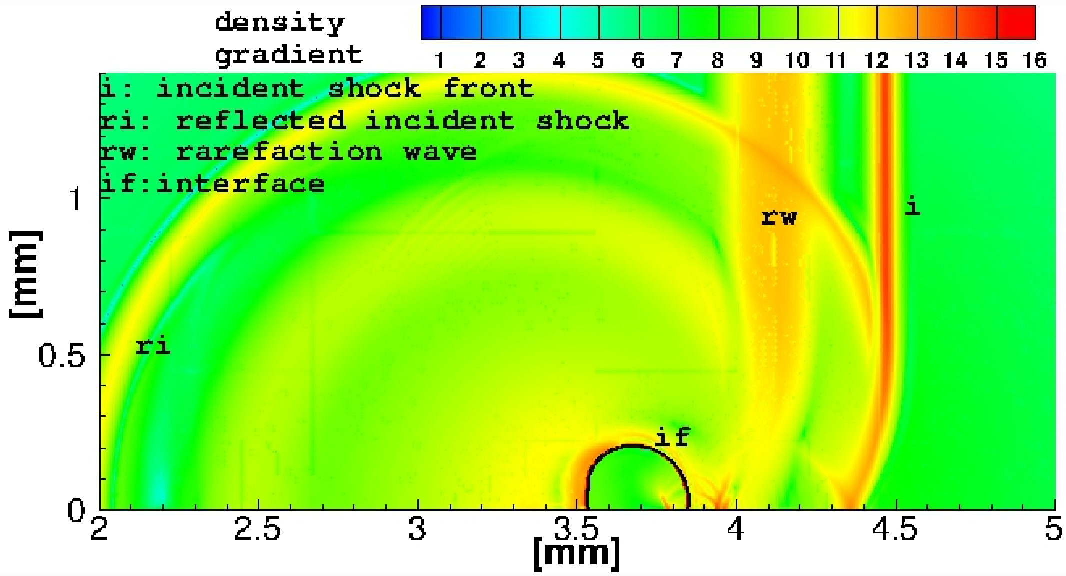


- Time: 44.09 μs

- Bubble diameter: 0.66 mm

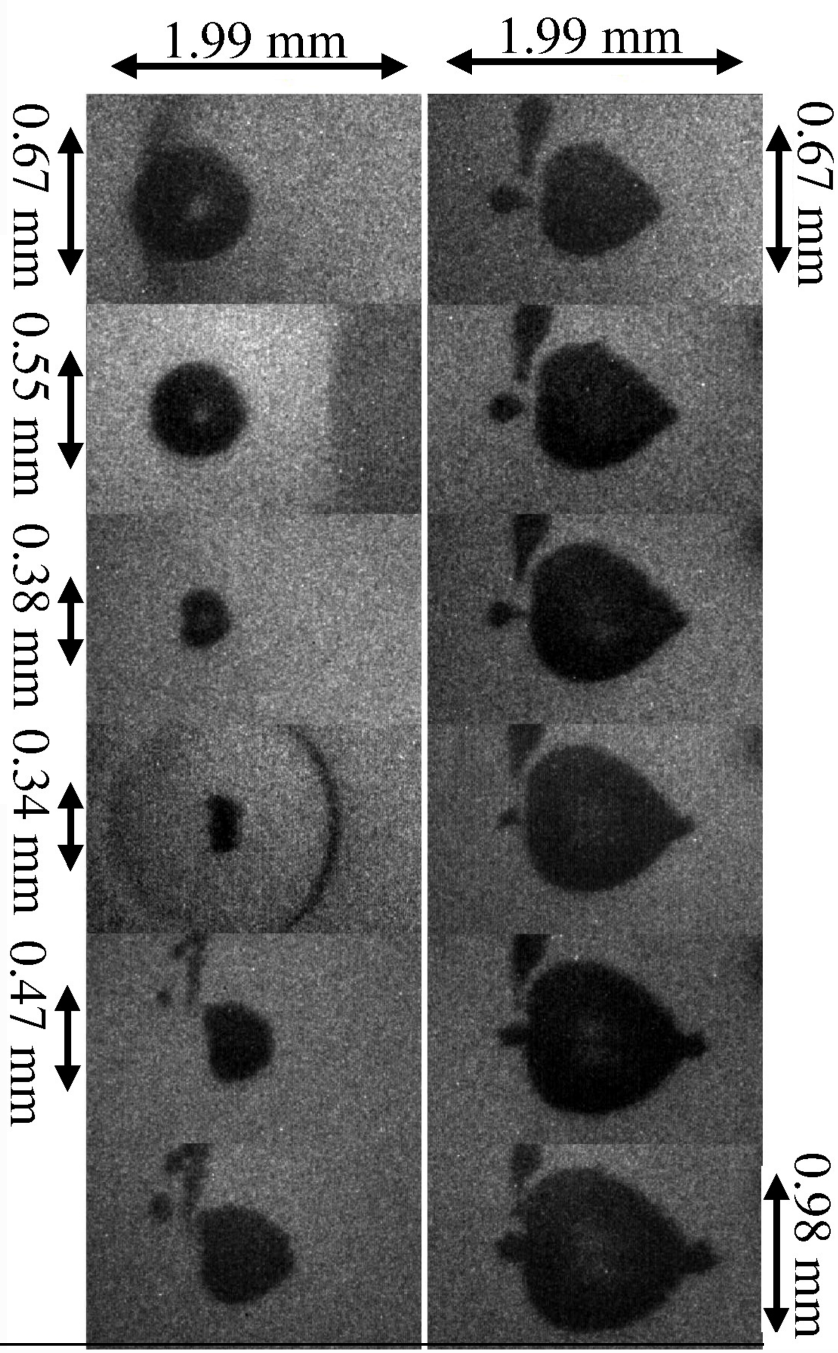
Numerical Result: Qualitative Comparison

- Collapsing bubble



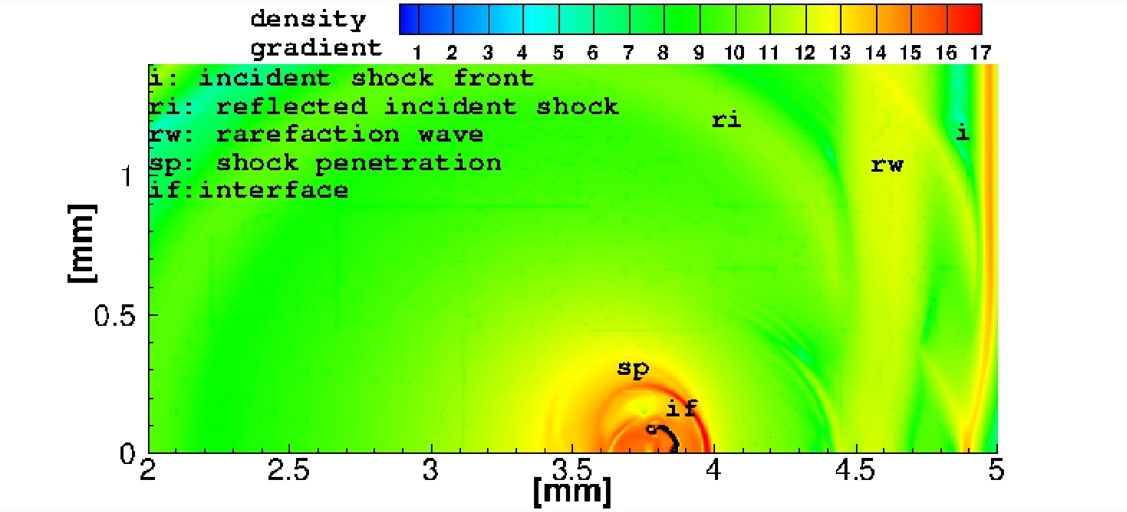
- Time: 44.52 μs

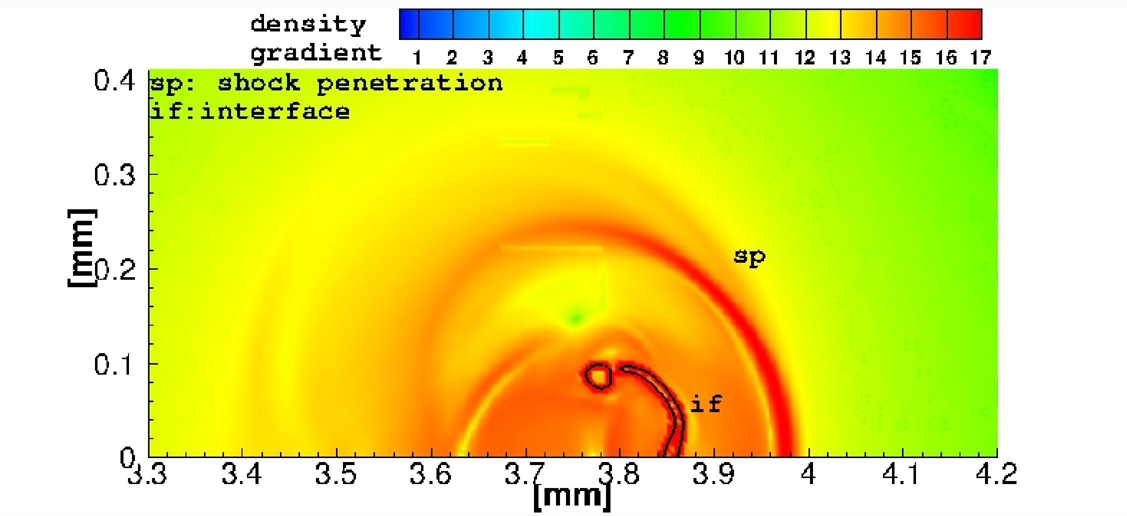
- Bubble diameter: 0.42 mm

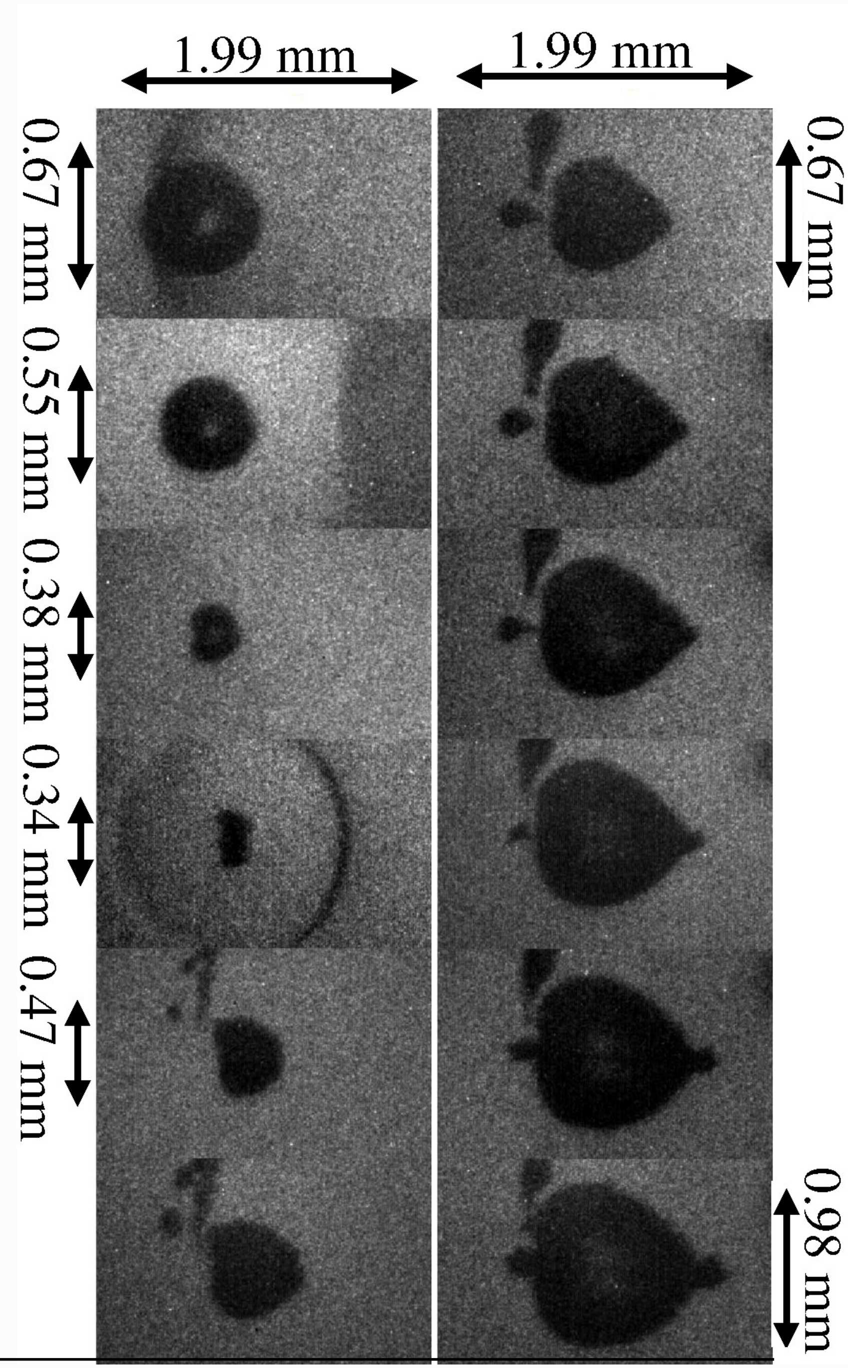


Numerical Result: Qualitative Comparison

- Penetration of the bubble



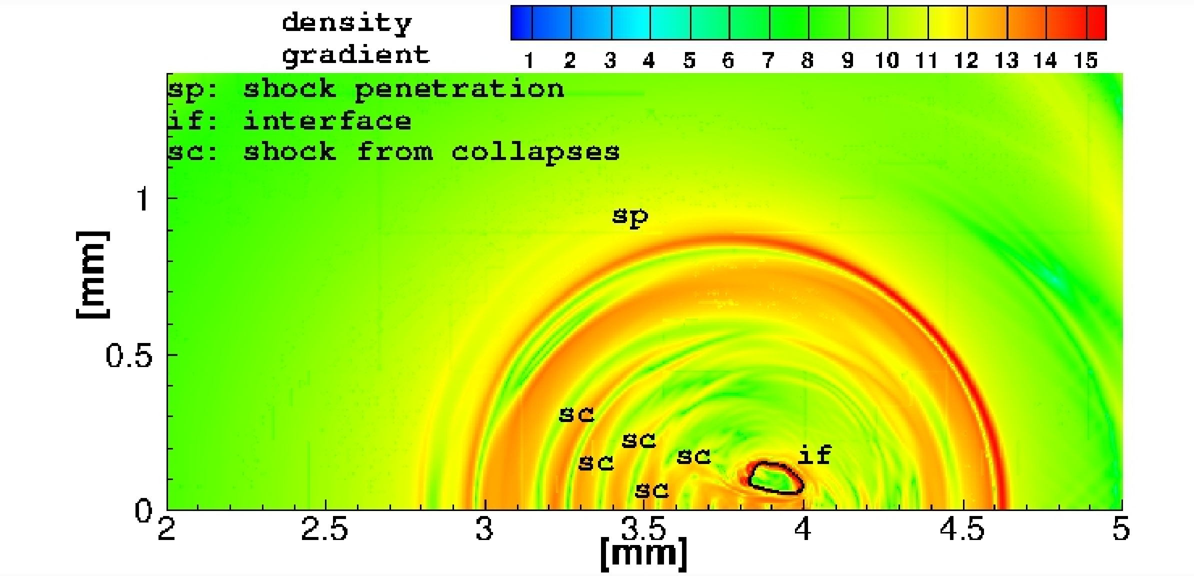


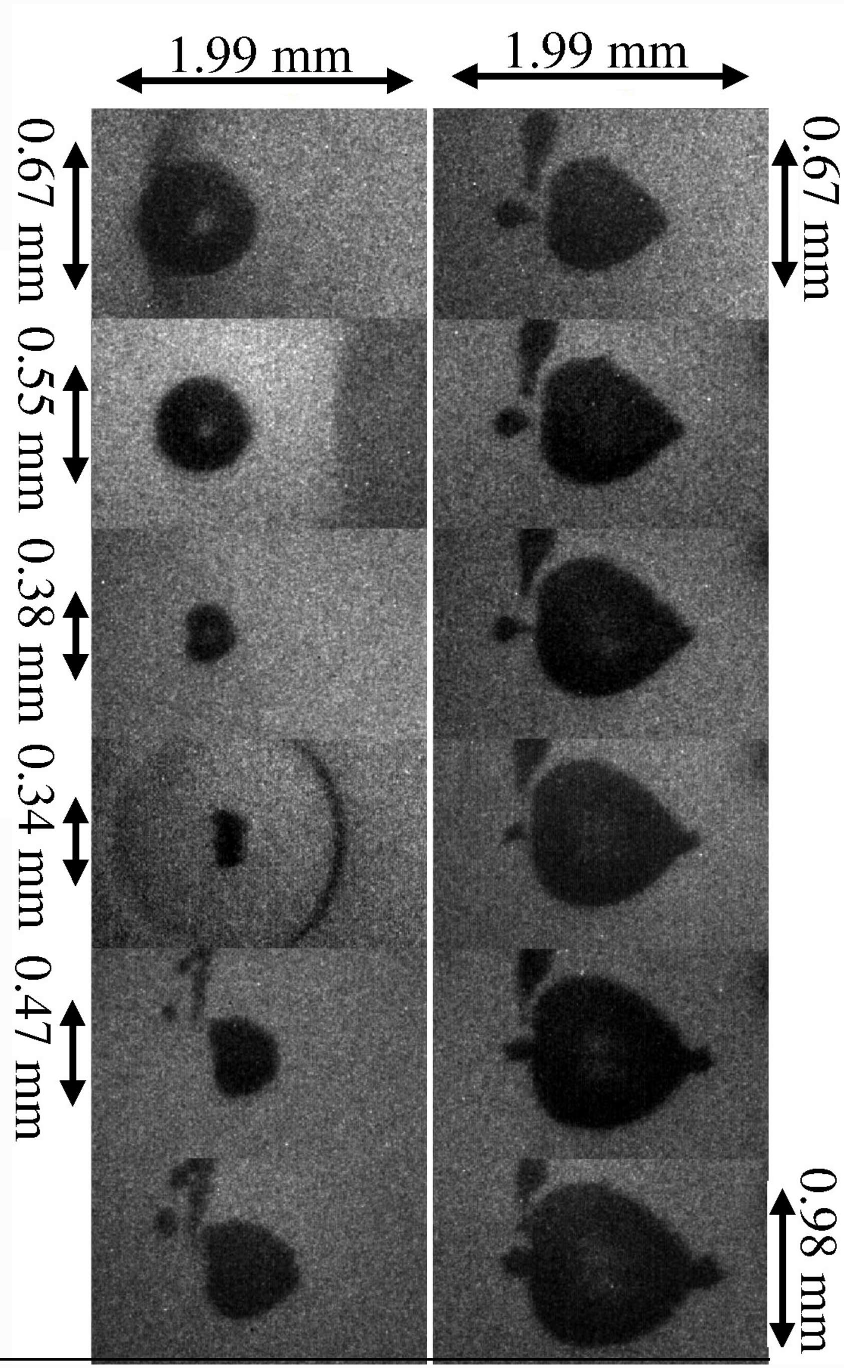


- Time: 44.85 μs. Bubble diameter: 0.2 mm

Numerical Result: Qualitative Comparison

- Propagation of the shock from the collapse

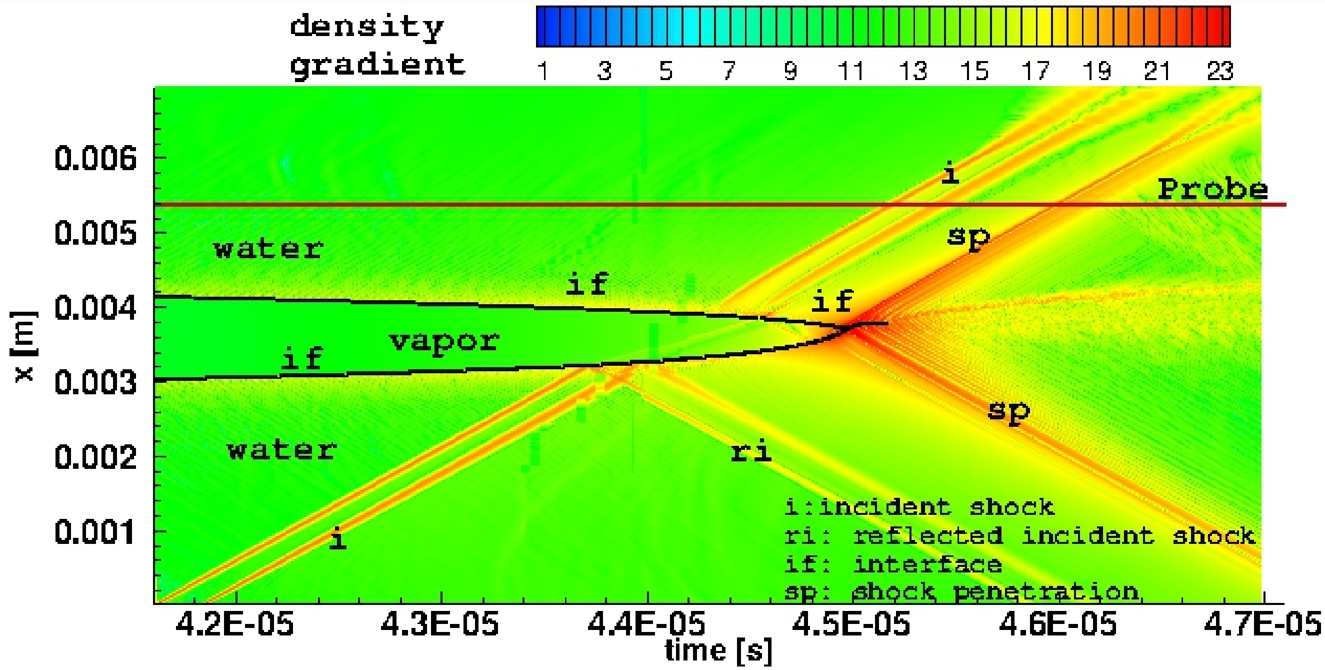


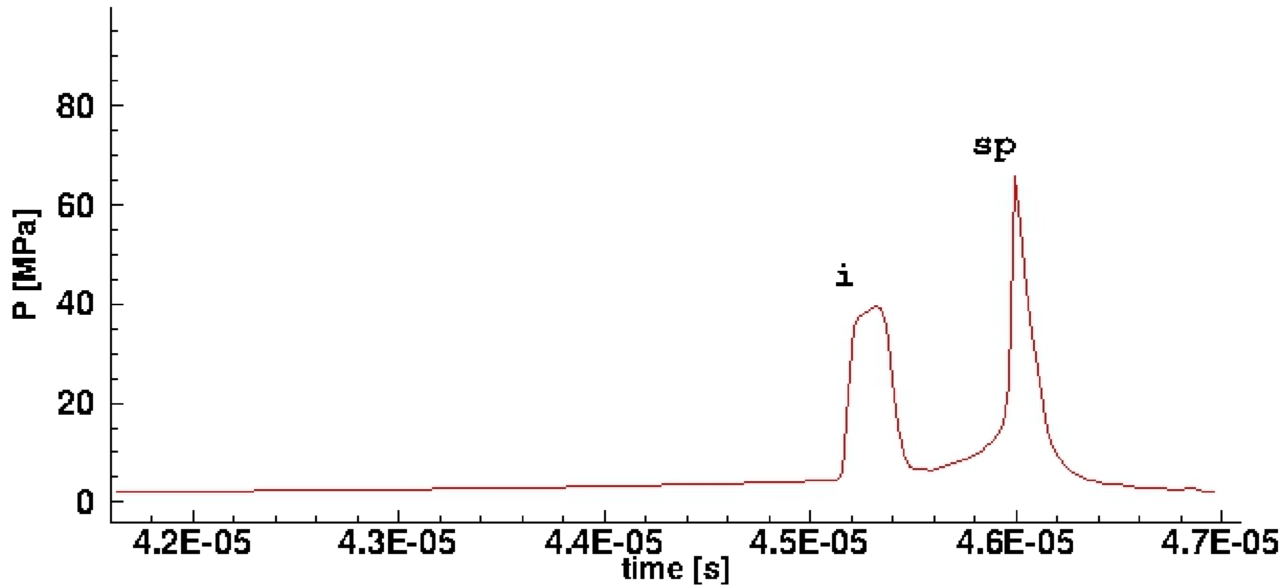


- Time: 45.25 μs

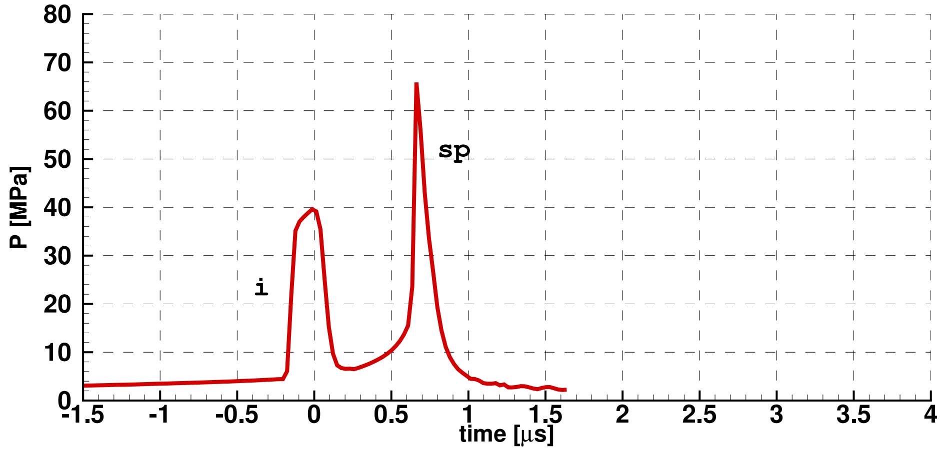
- Bubble diameter: 0.1 mm

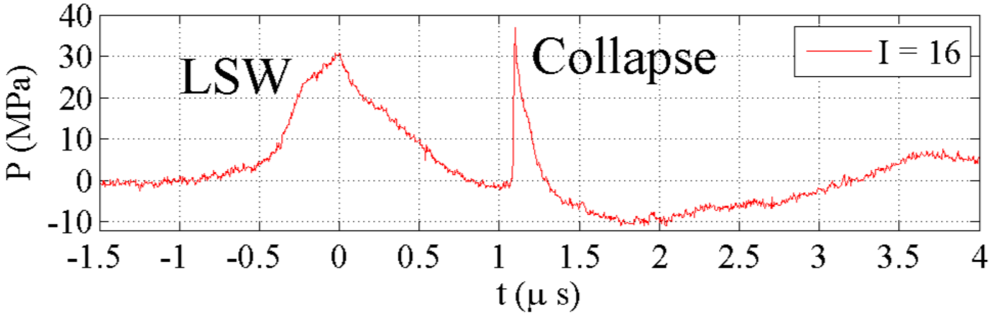
Numerical Result: Data along the Symmetry Line





Quantitative Comparison: Pressure Measurement after the Interaction





- $\Delta t = 0.67\,\mu$s
- First peak: $P = 40$ Mpa
- Second peak: $P = 66$ Mpa

- $\Delta t = 1.1\,\mu$s
- First peak: $P = 30$ Mpa
- Second peak: $P = 36$ Mpa

Conclusion

Qualitative comparison:

- Good agreement until collapse
  $\rightarrow$ the bubble size
  $\rightarrow$ penetration of the bubble
  $\rightarrow$ the shock from the collapse

- Limitations
  $\rightarrow$ the cloud of bubbles
  $\rightarrow$ resolution of tiny bubbles
  $\rightarrow$ quasi-2D computations

Quantitative comparison:

- Pressure amplitude depends on
  $\rightarrow$ the bubble size
  $\rightarrow$ the initial bubble state
  $\rightarrow$ the shock pulse