Model adaptation in hierarchies of hyperbolic systems

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Outline of the presentation

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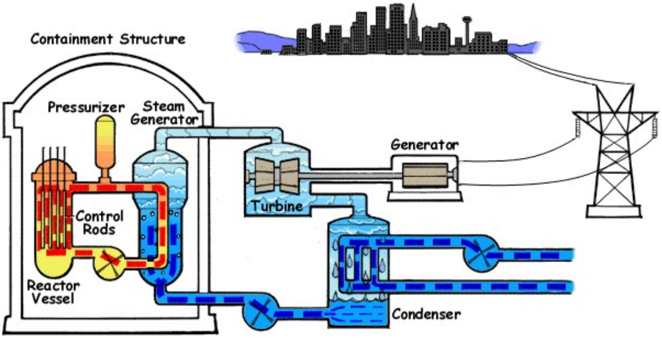
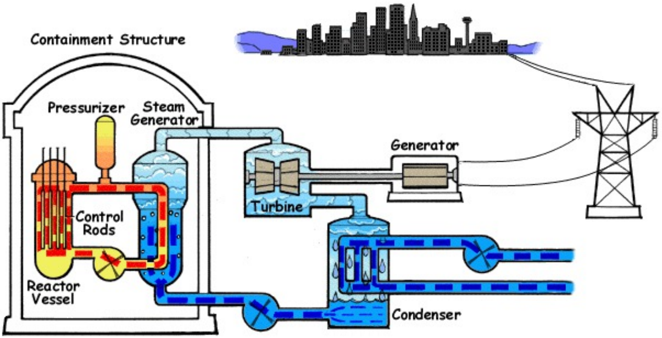
Simulation of the water circuit in a PWR



Context and difficulties

- Multiphase flows: Water liquid / water vapor / air
- Very heterogeneous flows and presence of tiny inclusions (droplets, bubbles...)
- Compressible effects (high temperature, high pressure...)

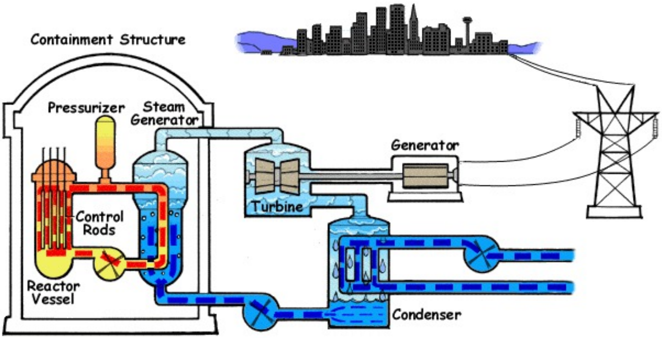
Simulation of the water circuit in a PWR



Modeling issues

- DNS impossible, use of a hierarchy of averaged models
- Different models according to the local scales and the accuracy of description
- Need of coupling between the different models

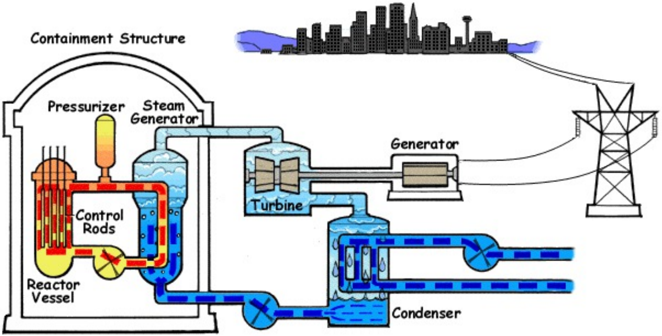
Simulation of the water circuit in a PWR



Difficulties in practice

- Understand the hierarchy of averaged models in an ideal case
- The models may have been developed independently
- Their compatibility is not ensured, even if the underlying Physics is the same!

Simulation of the water circuit in a PWR



Investigated models

- Compressible flows with high amplitudes (Euler systems)
- Averaged description (use of void fractions)
- Two-phase (or two-fluid) flows with exchanges

Coupling problems

LRC Manon (LJLL – CEA)
Modélisation et approximation numérique orientées pour l'énergie nucléaire

Different models to study:

- Different time/space scales, different regimes
  → hierarchy of models
- Formal connexion between models
  → asymptotic limits
- Differents for different codes
  → No exact compatibility

Ambroso, Boutin, Caetano, Cancès, Chalons, Coquel, Galié, Girardin, Godlewski, Kokh, Lagoutière, Mathis, Raviart, S. . .

Main developments:

- Asymptotic limits
- Interface coupling
- Optimization of the location of the coupling interface

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Main developments:

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- Optimization of the location of the coupling interface

Principle of the model adaptation

- [Cancès, Coquel, Godlewski, Mathis, S.] (see tomorrow morning)
- Project Osamoal of *Cemracs'11* ([Boulanger, Cancès, Mathis, Saleh, S.])

Given a fine model, the model adaptation consists in
the dynamic and automatic selection of the regions of the domain
where a coarser model can be used
(the coarse model being a "simplification" of the fine model)

GOAL: Optimization of the location of the coupling interface

Algorithm for the model adaptation

Algorithm.
Given u_0, $t^n = n\Delta t$ and a threshold Θ

- Two models to use
  - Fine model $\mathscr{M}_f$ with solution u_f
  - Coarse model $\mathscr{M}_c$ with solution u_c
- Partition $\mathbb{R}$ into $\mathscr{D}_f^n$ and $\mathscr{D}_c^n$ at each time step t^n
  - Indicator $\varepsilon(x, t^n) \sim \|u_f - u_c\|(x, t^{n+1})$
  - $\mathscr{D}_f^n := \{x \mid \varepsilon(x, t^n) > \Theta\}$ and $\mathscr{D}_c^n := \{x \mid \varepsilon(x, t^n) \leqslant \Theta\}$
- Solve the coupling problem between t^n and t^{n+1}
  - Solve $\mathscr{M}_f$ in $\mathscr{D}_f^n$
  - Solve $\mathscr{M}_c$ in $\mathscr{D}_c^n$
  - Coupling conditions at $\bar{\mathscr{D}}_f^n \cap \bar{\mathscr{D}}_c^n$

Works in progress

Two-phase flow models

- Toy models
- Euler equations
- Drift-Flux models
- Two-pressure two-velocity models

Asymptotic hierarchies

- Hyperbolic/hyperbolic relaxation
- Hyperbolic/parabolic relaxation
- 2(or 3)D/1D configurations

Indicators

- Error estimates and a posteriori estimates
- Chapman-Enskog expansions

Outline of the presentation

Context

Two-pressure two-velocity models for two-phase flows [Baer, Nunziato 89]

$$\begin{cases}\partial_t \alpha_1 + v_I(u)\partial_x \alpha_1 = \lambda_p(u)(p_1 - p_2) \\ \partial_t(\alpha_1\rho_1) + \partial_x(\alpha_1\rho_1 v_1) = -\Gamma \\ \partial_t(\alpha_2\rho_2) + \partial_x(\alpha_2\rho_2 v_2) = \Gamma \\ \partial_t(\alpha_1\rho_1 v_1) + \partial_x(\alpha_1\rho_1 (v_1)^2 + \alpha_1 p_1) - p_I(u)\partial_x\alpha_1 = \lambda_v(u)(v_2 - v_1) + f_1 \\ \partial_t(\alpha_2\rho_2 v_2) + \partial_x(\alpha_2\rho_2 (v_2)^2 + \alpha_2 p_2) - p_I(u)\partial_x\alpha_2 = \lambda_v(u)(v_1 - v_2) + f_2\end{cases}$$

Large relaxation coefficients λ_p and λ_v

Context

Two-pressure two-velocity models for two-phase flows [Baer, Nunziato 89]

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Large relaxation coefficients λ_p and λ_v
[Zuber, Findlay 65]: heuristic and empirical derivation

One pressure p, averaged velocity v and relative velocity v_r

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 \\ \partial_t(\rho Y) + \partial_x\big(\rho v Y + \rho Y(1-Y) v_r\big) = \Gamma \\ \partial_t(\rho v) + \partial_x\big(\rho v^2 + p + \rho Y(1-Y)(v_r)^2\big) = \rho(1-Y) f_1 + \rho Y f_2 \end{cases}$$

with $v_r = f(\rho, \rho Y, \rho v)$

Context

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Large relaxation coefficients λ_p and λ_v
[Ambroso, Chalons, Coquel, Galié, Godlewski, Raviart, S. 08]: asymptotic limits

Intermediate parabolic model [Guillard, Duval 07]

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 \\ \partial_t(\rho Y) + \partial_x\big(\rho v Y + \rho Y(1-Y)v_r\big) = \Gamma \\ \partial_t(\rho v) + \partial_x\big(\rho v^2 + p + \rho Y(1-Y)(v_r)^2\big) = \rho(1-Y)f_1 + \rho Y f_2 \end{cases}$$

with $v_r = f(\rho, \rho Y, \rho v, \partial_x p)$

Parabolic limit of hyperbolic systems

Study of numerical approximation, interface coupling and model adaptation for **parabolic limit of hyperbolic balance laws**

Worth change of behavior: difficult asymptotic limit (theory & numerics)

- smoothness
- boundary conditions (then coupling)
- CFL for explicit schemes
- All the others problems that we haven't encountered yet...

Theory: Marcati, Lattanzio, Yong, Coulombel... Al....
(forgetting about kinetic to Navier-Stokes equations!)

Our study

Project Osamoal of *Cemracs'11* ([Boulanger, Cancès, Mathis, Saleh, S.])
Optimized simulations by adapted models using asymptotic limits

- Goldstein-Taylor (or Telegraph) equations
- p-system with friction
- *Asymptotic preserving* schemes (compatible with the asymptotics)
- Interface coupling between hyperbolic balance laws and parabolic equations
- Indicators for the adaptation
- Model adaptation

Outline of the presentation

The Goldstein-Taylor equations

The Goldstein-Taylor equations

$$\begin{cases} \varepsilon\partial_t v + \partial_x u = 0, \\ \varepsilon\partial_t u + a^2\partial_x v = \dfrac{-\sigma}{\varepsilon} u, \end{cases} \qquad (\mathcal{M}_f^{GT})$$

where σ is a positive friction coefficient and a the sound speed.

Asymptotic limit: heat equation

$$\begin{cases} \partial_t v - \dfrac{a^2}{\sigma}\partial_{xx} v = 0, \\ u = 0. \end{cases} \qquad (\mathcal{M}_c^{GT})$$

The p-system with friction

The p-system with friction

$$\begin{cases} \varepsilon\partial_t \tau - \partial_x u = 0, \\ \varepsilon\partial_t u + \partial_x P(\tau) = \dfrac{-\sigma}{\varepsilon} u, \end{cases} \qquad (\mathcal{M}_f^{p-s})$$

where τ is the specific volume, u the velocity and σ is a positive friction coefficient. The function P is a classical pressure law.

Asymptotic limit: nonlinear heat equation

$$\begin{cases} \partial_t v + \dfrac{1}{\sigma}\partial_{xx} P(\tau) = 0, \\ u = 0. \end{cases} \qquad (\mathcal{M}_c^{p-s})$$

Outline of the presentation

Asymptotic preserving schemes

What does *asymptotic preserving* mean?

- Consistency wrt the asymptotic limit
- Stable for all regime (ie $\forall \varepsilon \geqslant 0$)
- Commutation of discretization ($\Delta x \to 0$) and of asymptotic ($\varepsilon \to 0$) limits

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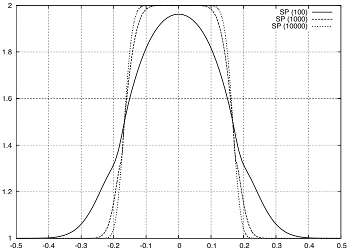
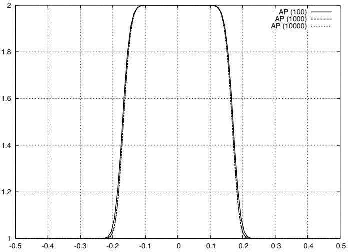


Splitting method



Asymptotic preserving schemes

[Chalons, Coquel, Godlewski, Raviart, S. 10] Euler + friction & gravity

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Asymptotic preserving schemes come from kinetic equations (linear PDE) [Jin et al. 98–99] [Gosse, Toscani 03]...

In the nonlinear context: [Enaux 07] [Buet, Franck, Després 10] [Berthon, Turpault 10] [Chalons et al. 10] [Berthon, LeFloch, Turpault 11]...
In general, only focus on consistency

Definition (The most restrictive...)

A numerical scheme for system ($\mathcal{M}_f$) is said to be asymptotic preserving if it is stable (under a CFL condition if necessary) and consistent with the solutions of ($\mathcal{M}_f$) for all $\varepsilon > 0$ and at the limit $\varepsilon \to 0$, it becomes a stable (under a CFL condition if necessary) and consistent with the solutions of ($\mathcal{M}_c$).

Here: hyperbolic to parabolic CFL condition for explicit schemes

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Here: hyperbolic to parabolic CFL condition for explicit schemes

The Goldstein-Taylor equations

- Godunov-type methods + Riemann solver involving the source term
- Riemann problem to solve à la LeRoux

$$\begin{cases} \varepsilon\partial_t v + \partial_x u = 0 \\ \varepsilon\partial_t u + a^2\partial_x v + \dfrac{\sigma}{\varepsilon} u\partial_x \xi = 0 \\ \partial_t \xi = 0 \end{cases}$$

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- Note $K = 1 + \frac{\sigma\Delta x}{2a\varepsilon}$

$$v_i^{n+1} = v_i^n - \frac{\Delta t}{\varepsilon K \Delta x}\big[\bar{u}(W_i^n, W_{i+1}^n) - \bar{u}(W_{i-1}^n, W_i^n)\big],$$
$$u_i^{n+1} = u_i^n - \frac{a^2\Delta t}{\varepsilon K \Delta x}\big[\bar{v}^-(W_i^n, W_{i+1}^n) - \bar{v}^+(W_{i-1}^n, W_i^n)\big],$$

where

$$\bar{u}(W_l, W_r) = \frac{u_l + u_r}{2} - \frac{a}{2}(v_r - v_l) \quad \bar{v}^{\pm}(W_l, W_r) = v_{l,r} \pm \frac{1}{a}\left(\frac{\bar{u}(W_l, W_r)}{K} - u_{l,r}\right)$$

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OK for the consistency...

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[Chalons, Coquel, Godlewski, Raviart, S. 10]...

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Pb of stability: $\Delta t \to 0$ when $\varepsilon \to 0$ to be stable (explicit Euler scheme)

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- Godunov-type methods + Riemann solver involving the source term
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$\Longrightarrow$ implicitation of the source term [Gosse, Toscani 03]

The Goldstein-Taylor equations

- Godunov-type methods + Riemann solver involving the source term
- Riemann problem to solve à la LeRoux
- Implicitation of the source term

Proposition ([Gosse, Toscani 03])

This numerical scheme is asymptotic preserving under the CFL condition

$$2\Delta t \leqslant \frac{\varepsilon \Delta x}{a} + \frac{\sigma \Delta x^2}{a^2}$$

- Hyperbolic to parabolic CFL
- Classical explicit 3-point scheme for the heat equation for $\varepsilon = 0$ (up to a numerical initial boundary layer)
- Theoretical proof of the commutation of the limits $\Delta x \to 0$ and $\varepsilon \to 0$ (decrease of the L^2 norm for all $\varepsilon \geqslant 0$)

The p-system with friction

- HLL scheme + linearized Riemann solver involving the source term
- Linearized Riemann problem to solve à la LeRoux
- Implicitation of the source term

Proposition ([Boulanger, Cancès, Mathis, Saleh, S. 12])

This numerical scheme is asymptotic preserving under the CFL condition

$$2\Delta t \leqslant \frac{\varepsilon \Delta x}{a} + \frac{\sigma \Delta x^2}{a^2}$$

where $a^2 \geq \sup_\tau (-P'(\tau))$ *(Whitham's condition)*

- Hyperbolic to parabolic CFL
- Classical explicit 3-point scheme for the nonlinear heat equation for $\varepsilon = 0$ (up to a numerical initial boundary layer)
- Theoretical proof of the commutation of the limits $\Delta x \to 0$ and $\varepsilon \to 0$ (entropy decreasing for all $\varepsilon \geqslant 0$)

Outline of the presentation

Hyperbolic/parabolic coupling

Between the fine model (hyperbolic system + relaxation) and the coarse model, we have to propose coupling conditions

In the parabolic regime $\varepsilon \ll 1$, we aim at recover a fully parabolic solution (the coupling interfaces are in “coarse” regions)

Hyperbolic/parabolic coupling

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Ambroso, Boutin, Caetano, Chalons, Coquel, Galié, Godlewski, Lagoutière, Raviart, S.. . . :

- Interface coupling for hyperbolic/hyperbolic problems
- Dirichlet boundary conditions
- Theory and numerics

Hyperbolic/parabolic coupling

Between the fine model (hyperbolic system + relaxation) and the coarse model, we have to propose coupling conditions

In the parabolic regime $\varepsilon \ll 1$, we aim at recover a fully parabolic solution (the coupling interfaces are in "coarse" regions)

BUT

$$\begin{cases} -\Delta u_l = 0 & x \in (-1,0) \\ -\Delta u_r = 0 & x \in (0,1) \\ u_l(-1) = u_r(1) = 0 \\ u_l(0^-) = u_r(0^+) \end{cases}$$

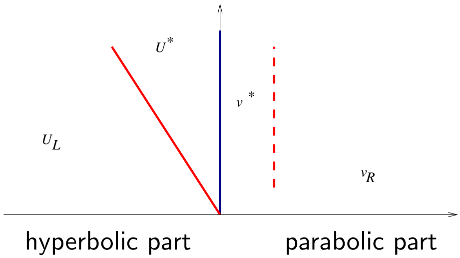
ILL-POSED!

Cure: add Neumann coupling condition...

Hyperbolic/parabolic coupling

Cure: add Neumann coupling condition $\rightarrow$ continuity of the flux [Boulanger, Cancès, Mathis, Saleh, S. 12]

- Use interfacial states on the left and on the right of the interface
- Solve the partial Riemann problem in the left-hand part
- Define the right interfacial state to obtain the parabolic flux
- Impose continuity of the fluxes of the conserved variable v



The Goldstein-Taylor equations

- Use interfacial states + continuity of v^*
- Solve the partial Riemann problem in the left-hand part
  $u^* - u_L = a(v_L - v^*)$
- Define the right interfacial state to obtain the parabolic flux
  $F_v^+ := -\frac{a^2}{\sigma} \frac{v_R - v^*}{\Delta x/2}$
- Impose continuity of the fluxes of the conserved variable v
  $F_v^- := u^*/\varepsilon \quad = F_v^+$

Then obtain

$$F_v = \left(\frac{1}{\varepsilon + \frac{\sigma \Delta x}{2a}} \right) (u_L + a(v_L - v_R))$$

$$F_u^- = \left(\frac{a^2}{\varepsilon + \frac{\sigma \Delta x}{2a}} \right) \left(\frac{\sigma \Delta x}{2\varepsilon a} v_L + v_R + \frac{\sigma \Delta x}{2\varepsilon a^2} u_L \right).$$

The p-system with friction

Do the same with a relaxation approximation at the left

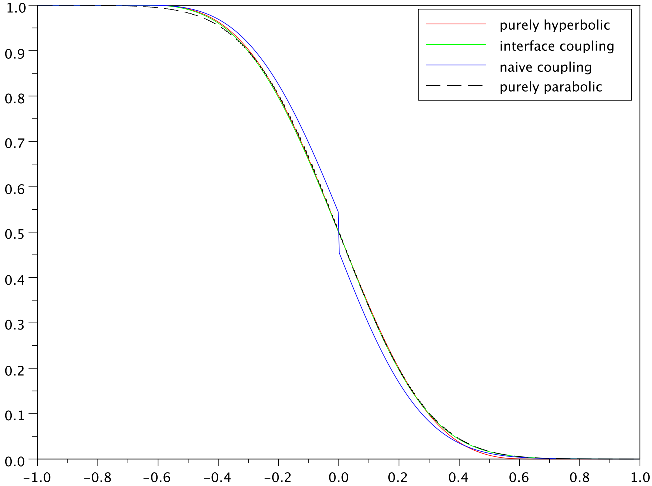
- Use interfacial states + continuity of the pressure π^*
- Solve the partial Riemann problem in the left-hand part
- Define the right interfacial state to obtain the parabolic flux
- Impose continuity of the fluxes of the conserved variable v

Then obtain

$$F_\tau = \frac{2}{\sigma\Delta x}\left[P(\tau_R) - \left(\frac{1}{1+\frac{\sigma\Delta x}{2\varepsilon a}}\right)\left(P(\tau_L) + \frac{2a\varepsilon}{\sigma\Delta x}P(\tau_R) + au_L\right)\right]$$

$$F_{u^-} = \left(\frac{1}{\varepsilon + \frac{2\varepsilon^2 a}{\sigma\Delta x}}\right)\left[P(\tau_L) + \frac{2a\varepsilon}{\sigma\Delta x}P(\tau_R) + au_L\right].$$

Numerical results for the Goldstein-Taylor



Outline of the presentation

Work in progress

- Generalisation of asymptotic preserving schemes to more complex models
- Generalisation of the hyperbolic + relaxation / parabolic interface coupling
- Numerical results...
- Indicators and adaptation...
- Theoretical study of the interface coupling [Golse, Salvarani 07]...
- Understanding of the full asymptotic "Baer-Nunziato models $\rightarrow$ Drift-flux models"