9th DFG-CNRS Workshop

Two-Phase Fluid Flows

Modeling, Analysis and Computational Methods

February 25 to 27, 2014

LABORATOIRE JACQUES-LOUIS LIONS

UNIVERSITE PIERRE ET MARIE CURIE

4 PLACE JUSSIEU, 75258 PARIS

TUESDAY afternoon chairman P.G. LeFloch	2:00pm—2:45pm **Robert Eymard**	2:45pm—3:30pm **Maren Hantke**	3:30pm Coffee break	4:00pm-4:45pm **Veronika Schleper**
WEDNESDAY morning chairman B. Boutin	10:00am—10h45 **Matthias Kotschote**	10:45–11:15am **M. Kränkel**	11:15am Coffee break	11:45–12:30 **Philippe Helluy**
WEDNESDAY afternoon chairman P. Helluy	2:30pm—3:15pm **Elisabetta Rocca**	3:15pm—3:45pm **M. Ndjinga**	3:45pm Coffee break	4:15pm-5:00pm **Jan Giesselmann**
THURSDAY morning chairman D. Kroener	10:00am—10h45 **Edwige Godlewski**	10:45–11:15am **C. Zeiler**	11:15am Coffee break	11:45–12h30 **Boris Andreianov**

Organizers

Benjamin Boutin (Rennes), Frédéric Coquel (Palaiseau), Philippe G. LeFloch (Paris)

INVITED SPEAKERS

Boris Andreainov (Besançon and Paris)

On quasi-Richards equation and finite volume approximation of two-phase flow with unlimited air mobility

Robert Eymard (Marnes-la-Vallée)

Some convergence results for numerical schemes approximating two phase flow in porous media

Jan Giesselmann (Stuttgart)

A posteriori analysis of a discontinuous Galerkin scheme for a diffuse interface model

Edwige Godlewski (Paris)

Some tools for model adaptation in the context of fluid flows

Maren Hantke (Magdeburg)

Compressible Euler equations for two phase flow with phase transition

Philippe Helluy (Strasbourg)

Two-fluid pressure laws and non-convex hyperbolic domains (joint with J. Jung)

Matthias Kotschote (Konstanz)

Well-posedness results for compressible two-phase models

Elisabetta Rocca (Berlin)

Non-isothermal two phase flows of incompressible fluids

Veronika Schleper (Stuttgart)

Coupling of compressible and incompressible fluids

CONTRIBUTING SPEAKERS

Mirko Kränkel (Freiburg)

Discontinuous Galerkin schemes for the compressible Navier-Stokes-Allen-Cahn system

Michaël Ndjinga (Gif-Sur-Yvette)

The Riemann problem for an incompressible two-fluid model

Christoph Zeiler (Stuttgart)

Sharp interface approach for liquid-vapor flow with phase transition

LIST of ABSTRACTS

Boris Andreianov (Besançon and Paris) ***On quasi-Richards equation and finite volume approximation of two-phase flow with unlimited air mobility***

Abstract. Is the Richards equation the correct limit of the two-phase flow equation when the mobility \mu of the air phase goes to infinity ? In a recent work of Eymard, Henry and Hilhorst, a "quasi-Richards" model was deduced as the relevant singular limit. We get partial uniqueness results for this model and show, using the approach of renormalized solutions of Gagneux and Plouvier-Debaigt, that Richards and quasi-Richards models coincide in absence of source terms. Let us stress that we do not expect that the two models coincide in general. Besides theoretical issues, the main focus of the talk is on the robust, with respect to the air mobility parameter mu, finite volume approximation of the two-phase model. The scheme we propose is "almost" asymptotic preserving, it is convergent for every mu, and numerical experiments show that it behaves well even for much higher values of mu than those encountered in practice. The talk is based on works in collaboration with R. Eymard, M. Ghilani and N. Marhraoui.

Robert Eymard (Marne-la-Vallée) ***Some convergence results for numerical schemes approximating two phase flow in porous media***

Abstract. This talk is focused on the mathematical study of numerical schemes, devoted to the approximation of two-phase flow in porous media. The main difficulty in this study is the difference between the continuous setting, where a large class of test functions is available, and the discrete one, where the discrete space operators are limiting the possibilities for choosing test functions. This difficulty is explored on different examples of two-phase flow models. In some of them, convergence proofs are only completed in the case of two-point flux approximation for the diffusion terms.

Jan Giesselmann (Stuttgart) ***A posteriori analysis of a discontinuous Galerkin scheme for a diffuse interface model***

Abstract. We study the numerical approximation of a one dimensional van-der- Waals fluid by a semi-discrete discontinuous Galerkin scheme. In particular, we will derive an a posteriori error estimate. The derivation is based on two steps. Firstly, we will describe a modification of the classical relative entropy framework in which the regularization compensates for the non-convex energy, thereby offering a stability framework for the model at hand. Secondly, we will present a reconstruction of the numerical solution which satisfies a perturbed version of the original problem and is regular enough such that the modified relative entropy framework can be employed. This is joint work with Ch. Makridakis (Univ. of Sussex) and T. Pryer (Univ. of Reading).

Edwige Godlewski (Paris) ***Some tools for model adaptation in the context of fluid flows***

Abstract. In the context of multiphase flow, the phenomena one wants to simulate can be described by several models, according to the main features of the flow and to the accuracy of the description which is needed in the simulation. Then one is interested in choosing the right model, coarse or fine, according to the desired accuracy and switching from one model to the other requires not only coupling the two models at some interface, but also a model adaptation procedure. In order to be able to define and analyze a precise model adaptation method, we consider a theoretical one dimensional framework where the two models are linked by a relaxation process. We construct a numerical indicator which, given some tolerance level, enables to determine the intervals of the computational domain where the relaxation fine model can be replaced by the equilibrium coarse one. We also consider a 2x2 system and the corresponding equilibrium scalar conservation law to obtain some precise error estimates in order to validate the approach, even if on a toy model.

Philippe Helluy (Strasbourg) ***Two-fluid pressure laws and non-convex hyperbolic domains***

Abstract. We consider a two-fluid compressible flow. Each fluid obeys a stiffened gas pressure law. The continuous model is well defined without considering mixture regions. However, for numerical applications it is often necessary to consider artificial mixtures, because the two-fluid interface is diffused by the numerical scheme. We show that classic pressure law interpolations lead to non-convex hyperbolicity domain and failure of well-known numerical schemes. We propose a physically relevant pressure law interpolation construction and show that it leads to a necessary modification of the pure phase pressure laws. We also propose a numerical scheme that permits to approximate the stiffened gas model without artificial mixture. This is a joint work with J. Jung.

Matthias Kotschote (Konstanz) ***Well-posedness results for compressible two-phase models***

Abstract. In this talk two different diffuse interface models are presented. On the one hand we are getting acquainted with the "Navier-Stokes-Allen-Cahn" (NSAC) system, a combination of the compressible Navier-Stokes equations with a Allen-Cahn phase field description that admits of describing two-phase patterns in a flowing liquid including phase transformations. On the other this model finds its counterpart in the "Navier-Stokes-Cahn-Hilliard" system that imitates again a flow of a binary mixture being macroscopically immiscible but without permitting phase transformation. One main objective is to present existence and uniqueness of local (in time) strong solutions of these systems, but some physical background and possible situations, e.g. mixture of compressible and incompressible fluids, of these models will be provided as well.

Elisabetta Rocca (Berlin) ***Non-isothermal two phase flows of incompressible fluids***

Abstract. We introduce a diffuse interface model describing the evolution of a mixture of two different viscous incompressible fluids of the same density. The effects of temperature on the flow are taken into account. In the mathematical model, the evolution of the velocity is ruled by the Navier-Stokes system with temperature-dependent viscosity, while the order parameter representing the concentration of one of the components of the fluid is assumed to satisfy a convective Cahn-Hilliard equation. The effects of the temperature are prescribed by a suitable form of the heat equation. However, due to quadratic forcing terms, this equation is replaced, in the weak formulation of the model, by an equality representing energy conservation complemented with a differential inequality describing production of entropy. Global-in-time existence for the initial-boundary value problem associated to the weak formulation of the model is proved by deriving suitable a-priori estimates and showing weak sequential stability of families of approximating solutions. This is a joint work with Michela Eleuteri (University of Milano) and Giulio Schimperna (University of Pavia).

Veronika Schleper (Stuttgart) ***Coupling of compressible and incompressible fluids***

Abstract. We discuss the limit of compressible to incompressible Euler equations in the context of two-phase flows. Hereby, the limit is obtained in the liquid phase, while the compressibility of the gas phase is preserved. In a first step, we show how weak solutions of the fully compressible two-fluid model depend on the sound speed in the liquid phase. From these observations, we construct a set of initial data, such that the solution to the fully compressible model exists for all (sufficiently large) liquid sound speeds. This result at hand, we can go to the incompressible limit in the liquid phase and obtain the convergence of the semigroup of fully compressible solutions to the semigroup constructed by a coupling of compressible and incompressible Euler equations.

CONTRIBUTING SPEAKERS

Mirko Kränkel (Freiburg) ***Discontinuous Galerkin schemes for the compressible Navier-Stokes-Allen-Cahn system***

Michaël Ndjinga (Gif-Sur-Yvette) ***The Riemann problem for an incompressible two-fluid model***

Abstract. The transition between two-phase and single-phase flows in the cooling system of nuclear power plants raises many challenging theoretical and numerical difficulties. Standard numerical schemes applied to two-fluid models perform poorly in the regime where one of the phase is nearly absent. The positivity of some physical parameters is not guaranteed and instabilities may appear during the numerical simulation. In order to better understand this problem and design robust numerical solvers we first study the incompressible limit of our model. The incompressible model is composed of two conservation laws and the characteristic fields are neither genuinely non linear, nor linearly degenerate. This yields an original wave structure in the Riemann problem with configurations including three waves such as 2-wave 1-wave 2-wave or 1-wave 2-wave 1-wave. We however prove the existence and uniqueness of an admissible solution, which enables us to design positive Riemann solvers able to simulate pure phase regimes.

Christoph Zeiler (Stuttgart) ***Sharp interface approach for liquid-vapor flow with phase transition***

Abstract. We consider a mathematical model that describes the dynamics of fluids which can occur in a liquid and a vapor phase. The phase boundary is represented as a sharp, shock like interface. Surface tension and the generalised Gibbs-Thomson law control mass flux across the interface and the amount of entropy dissipation. We propose a moving mesh strategy to solve the problem numerically what requires the solution of Riemann-like (microscale) problems across the interface. The essential difficulty, solving such Riemann-like problems, is the correct treatment of mass flux and to overcome problems posed by the underlying mixed hyperbolic-elliptic type model. We present exact Riemann solvers for kinetic relations based on (a) the Gibbs-Thomson law and (b) the Liu entropy criterion. Numerical examples in one and two spacial dimensions and pressure laws of real fluids demonstrate the applicability of the sharp interface approach.

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